Lecture 4: Network Security Principles

-COMP 6712 Advanced Security and Privacy

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Network Security Principles

Recall RSA and Digital Signature

Authenticated Key Exchange

Public Key Infrastructure(PKI)

and Certification Authorities

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Public key encryption

Diffie-Hellman

$$G = \langle g \rangle$$

$$a \overset{\$}{\leftarrow} \{1, \dots, |G|\}$$

$$A \leftarrow g^{a}$$

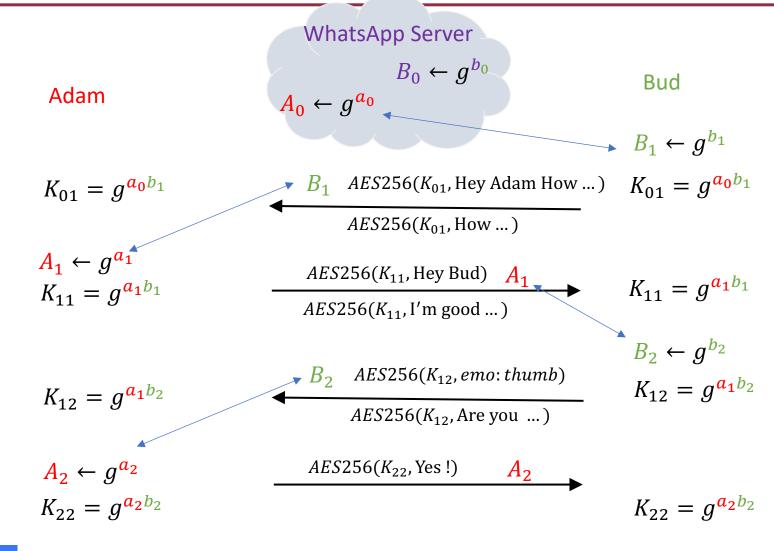
$$B \leftarrow g^{b}$$

$$K \leftarrow B^{\mathbf{a}} = g^{\mathbf{a}b}$$

$$K \leftarrow \mathbf{A}^b = g^{\mathbf{a}b}$$

Ratchet Diffie-Hellman in WhatsApp and Signal







ElGamal

ElGamal. Enc : $G \times G \rightarrow G \times C$

$$G = \langle g \rangle$$

ElGamal. Dec : $\mathbf{Z}_p \times G \times G \rightarrow G$

A, C

KeyGen

1.
$$sk = b \stackrel{\$}{\leftarrow} \{1, \dots, |G|\}$$

2.
$$pk = B \leftarrow g^k$$

3. return (sk, pk)

Enc(pk, M)

1.
$$a \stackrel{\$}{\leftarrow} \{1, ..., |G|\}$$

2.
$$A \leftarrow g^a$$

3.
$$K \leftarrow B^a = g^{ab}$$

4.
$$C \leftarrow K \cdot M$$

return (A, C)

Dec(sk, C)

1.
$$Z \leftarrow A^b = g^{ab}$$

2. $M \leftarrow C/K$

2.
$$M \leftarrow C/K$$

return M

RSA in 1977

The RSA encryption scheme

$$c = E(m) = m^e \pmod{n}$$



Adi Shamir

Ron Rivest

Leonard Adleman

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Euler's Theorem

Theorem: if (G, \circ) is a finite group, then for all $g \in G$:

$$g^{|G|} = e$$

•
$$(\mathbf{Z}_p^*, \cdot)$$
: $|\mathbf{Z}_p^*| = (p-1)$ $e=1$

Fermat's theorem: if p is prime, then for all $a \neq 0 \pmod{p}$:

$$a^{p-1} \equiv 1 \pmod{p}$$

• (Z_n^*, \cdot) : $|Z_n^*| = \phi(n)$ e = 1

Euler's theorem: for all positive integers n, if gcd(a, n) = 1 then

$$a^{\phi(n)} \equiv 1 \pmod{n}$$

Structure for RSA

$$n\leftarrow p\cdot q$$
 $\phi(n)=(p-1)(q-1)$ $Z_n^*=\text{invertible elements in }Z_n=\{a\in Z_n\mid\gcd(a,n)=1\}$ $(Z_n^*,\cdot)\text{ is a group of order }\phi(n)!$ $a^{\phi(n)}\equiv 1\pmod n$ $ed=1\mod\phi(n)$

RSA Enc
$$M \xrightarrow{\textbf{Enc}} C \leftarrow M^e \mod n \xrightarrow{\textbf{Dec}} M \leftarrow C^d \mod n$$

Textbook RSA

$$\mathcal{PK}$$
 \mathcal{M}

RSA. Enc:
$$\mathbf{Z}^+ \times \mathbf{Z}_{\phi(n)}^* \times \mathbf{Z}_n^* \rightarrow \mathbf{Z}_n^*$$

RSA. Dec:
$$\mathbf{Z}_{\phi(n)}^* \times \mathbf{Z}_n^* \rightarrow \mathbf{Z}_n^*$$
 $\mathcal{SK} \quad \mathcal{C} \quad \mathcal{M}$

$$\mathbf{Enc}(pk = (n, e), M \in \mathbf{Z}_n^*)$$

- $C \leftarrow M^e \mod n$
- return C

Common choices of e: 3, 17, 65 537 $11_2 \quad 10001_2 \quad 10000000000000001_2$

KeyGen

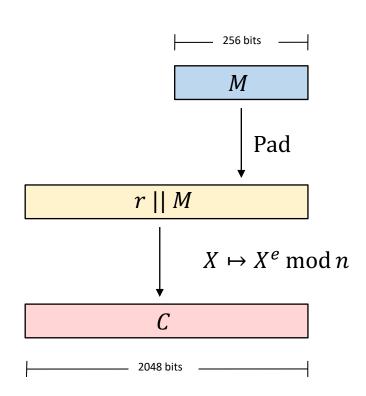
- $p, q \leftarrow \text{two random prime numbers}$
- $\phi(n) = (p-1)(q-1)$
- 4. **choose** e such that $gcd(e, \phi(n)) = 1$
- 5. $d \leftarrow e^{-1} \mod \phi(n)$
- 6. $sk \leftarrow d$ $pk \leftarrow (n, e)$
- return (sk, pk)

$\mathbf{Dec}(sk = d, C \in \mathbf{Z}_n^*)$

- $M \leftarrow C^d \mod n$
- return M

RSA in practice

- Textbook RSA is deterministic ⇒ cannot be IND-CPA secure
- How to achieve IND-CPA, IND-CCA?
 - pad message with random data before applying RSA function
 - PKCS#1v1.5 (RFC 2313)
 - RSA-OAEP (RFC 8017)
- Do not use Textbook RSA
- RSA encryption not used much in practice anymore



A short summary

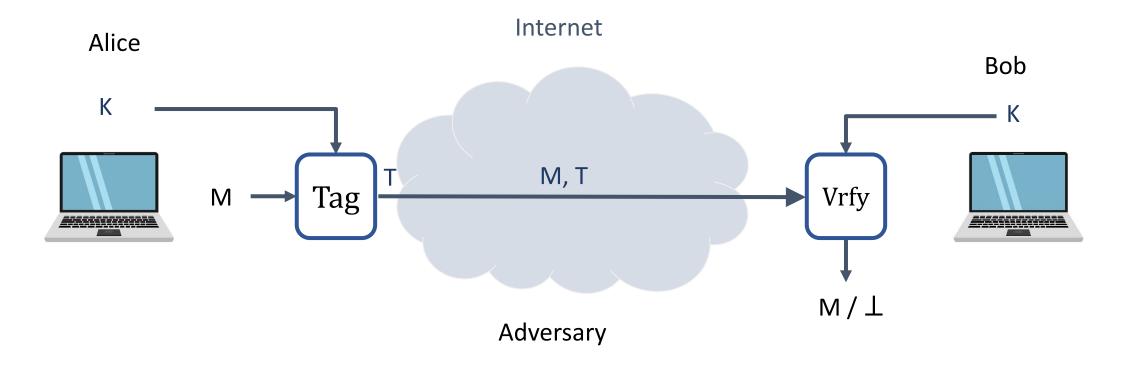
• We can build IND-CPA secure ElGamal scheme based on DDH assumption

 Padding with randomness, we can transfer Textbook RSA to IND-CPA scheme

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Digital Signature

Achieving integrity: MACs

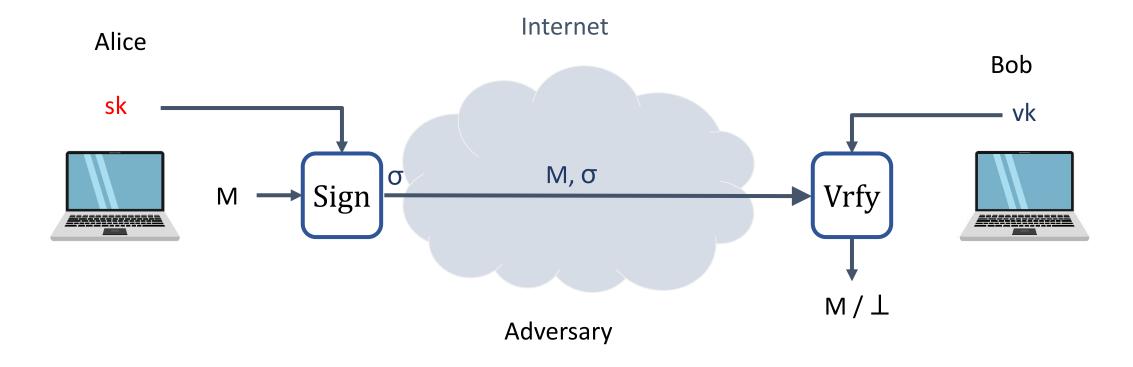


Tag: tagging algorithm (public)

K: tagging / verification key (secret)

Vrfy: verification algorithm (public)

Achieving integrity: digital signatures



Sign: tagging algorithm (public)

Vrfy: verification algorithm (public)

sk : signing key (secret)

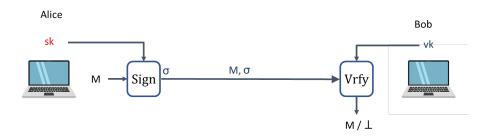
vk : verification key (public)

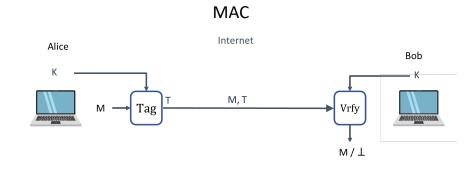
Digital signatures vs. MACs

• Digital signatures can be verified by anyone

 MACs can only be verified by party sharing the same key

Digital signature





- Non-repudiation: Alice cannot deny having created σ
 - But she can deny having created *T* (since Bob could have done it)

Digital signatures — syntax

A **digital signature** scheme is a tuple of algorithms $\Sigma = (\text{KeyGen, Sign, Vrfy})$

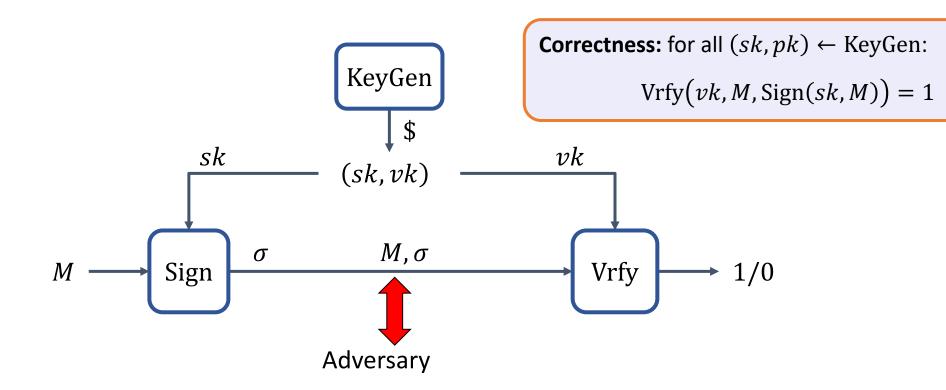
KeyGen : () $\rightarrow S\mathcal{K} \times \mathcal{V}\mathcal{K}$

$$Sign: \mathcal{SK} \times \mathcal{M} \to \mathcal{S}$$

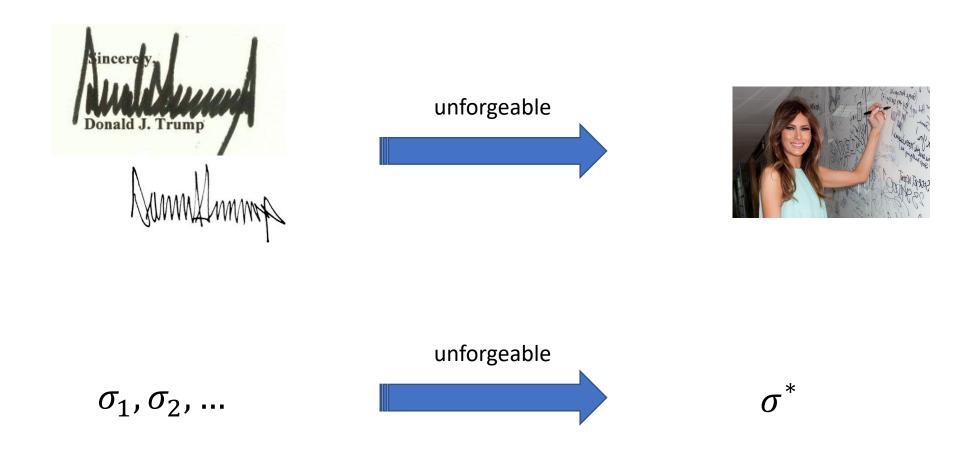
$$\operatorname{Sign}(sk, M) = \operatorname{Sign}_{sk}(M) = \sigma$$

 $Vrfy: \mathcal{VK} \times \mathcal{M} \times \mathcal{S} \rightarrow \{0,1\}$

$$\operatorname{Sign}(sk, M) = \operatorname{Sign}_{sk}(M) = \sigma$$
 $\operatorname{Vrfy}(vk, M, \sigma) = \operatorname{Vrfy}_{vk}(M, \sigma) = 1/0$



Signature: unforgeability



$$n \leftarrow p \cdot q$$

$$n \leftarrow p \cdot q$$

$$\phi(n) = (p-1)(q-1)$$

$$Z_n^* = \text{invertible elements in } Z_n = \{ a \in Z_n \mid \gcd(a,n) = 1 \}$$

$$(Z_n^*, \cdot) \text{ is a group of order } \phi(n)!$$

$$a^{\phi(n)} \equiv 1 \pmod{n}$$
 $ed = 1 \mod{\phi(n)}$

RSA Enc
$$M \xrightarrow{\textbf{Enc}} C \leftarrow M^e \mod n \xrightarrow{\textbf{Dec}} M \leftarrow C^d \mod n$$

Textbook RSA signatures

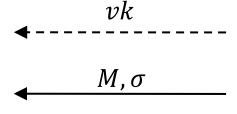
RSA. Sign:
$$\mathbf{Z}^+ \times \mathbf{Z}_{\phi(n)}^* \times \mathbf{Z}_n^* \rightarrow \mathbf{Z}_n^*$$

RSA. Vrfy:
$$\mathbf{Z}^+ \times \mathbf{Z}_{\phi(n)}^* \times \mathbf{Z}_n^* \times \mathbf{Z}_n^* \to \{1,0\}$$

$\mathbf{Vrfy}(vk = (n, e), M \in \mathbf{Z}_n^*, \sigma)$

- 1. **if** $\sigma^e = M \mod n$ **then**
- 2. return 1
- 3. else
- 4. return 0







KeyGen

- 1. $p, q \leftarrow \text{two random prime numbers}$
- 2. $n \leftarrow p \cdot q$
- 3. $\phi(n) = (p-1)(q-1)$
- 4. **choose** e such that $gcd(e, \phi(n)) = 1$
- 5. $d \leftarrow e^{-1} \mod \phi(n)$
- 6. $sk \leftarrow (n, d)$ $vk \leftarrow (n, e)$
- 7. **return** (sk, vk)

$\mathbf{Sign}(sk = (n, d), M \in \mathbf{Z}_n^*)$

- 1. $\sigma \leftarrow M^d \mod n$
- 2. return σ

 $d = e^{-1} \bmod \phi(n) \iff ed = 1 \bmod \phi(n)$

$$\sigma^e = M^{de} = M^{ed \mod \phi(n)} = M^1 = M \mod n$$

Insecurity of Textbook RSA signature

Given
$$\sigma_1 = M_1^d$$
, $\sigma_2 = M_2^d$

$$\sigma_1 \sigma_2 = (M_1 M_2)^d mod n$$
 is a signature of $M_1 M_2 mod n$

Many other attacks exist

RSA-FDH: Hash-then sign paradigm

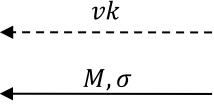
RSA. Sign:
$$\mathbf{Z}^+ \times \mathbf{Z}^*_{\phi(n)} \times \{0,1\}^* \to \mathbf{Z}^*_n$$

RSA. Vrfy:
$$\mathbf{Z}^+ \times \mathbf{Z}_{\phi(n)}^* \times \{0,1\}^* \times \mathbf{Z}_n^* \to \{1,0\}$$

$\mathbf{Vrfy}(vk = (n, e), M \in \mathbf{Z}_n^*, \sigma)$

- 1. **if** $\sigma^e = H(M) \mod n$ **then**
- 2. return 1
- 3. else
- 4. return 0







$$H: \{0,1\}^* \to \mathbf{Z}_n^*$$

KeyGen

- 1. $p, q \leftarrow \text{two random prime numbers}$
- 2. $n \leftarrow p \cdot q$
- 3. $\phi(n) = (p-1)(q-1)$
- 4. **choose** e such that $gcd(e, \phi(n)) = 1$
- 5. $d \leftarrow e^{-1} \mod \phi(n)$
- 6. $sk \leftarrow (n, d)$ $vk \leftarrow (n, e)$
- 7. **return** (sk, vk)

$\mathbf{Sign}(sk = (n, d), M \in \mathbf{Z}_n^*)$

- 1. $\sigma \leftarrow H(M)^d \mod n$
- σ return σ

RSA-FDH: Hash-then sign paradigm

Theorem: For any UF-CMA adversy A against hashed RSA making q SIGN $_{sk}(\cdot)$ queries, there is an algorithm B solving the RSA-problem:

$$\mathbf{Adv}_{\mathrm{RSA},H}^{\mathrm{uf-cma}}(A) \leq q \cdot \mathbf{Adv}_{n,e}^{\mathrm{RSA}}(B)$$

where H is assumed perfect*

^{*} H is assumed to be random oracle, which is out of the scope of this course. Refer to [KL] Section 12.4 for the formal proof

From the view of attack

Given
$$\sigma_1 = H(M_1)^d$$
, $\sigma_2 = H(M_2)^d$

$$\sigma_1 \sigma_2 = (H(M_1)H(M_2))^d \mod n$$
 is a signature of some m ??

Find m such that $H(m) = H(M_1)H(M_2)!!!!!$ One-wayness of H

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Digital signature using in practice

RSA signature

• RSAwithSHA-256,382,512

(PKCS #1 V2.1, RFC 6594)

ECDSA signature

• ECDSA256,384,512

• EdDSA

(NIST FIPS 186-4) (RFC 6979)

Schnorr signature

A short summary

• Hash-then sign paradigm of RSA gives a secure signature

• There are Discrete-log-based signatures, ECDSA, and Schnorr

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Primitive	Functionality + syntax	Hardness assumption	Security	Examples
Diffie-Hellman	Derive shared value (key) in a cyclic group $A^b = g^{ab} = B^a$	Discrete logarithm (DLOG) Decisional Diffie-Hellman (DDH)		$ig(oldsymbol{Z}_p^*,\cdotig)$ $-$ DH $ig(Eig(oldsymbol{F}_pig),+ig)$ $-$ DH
RSA function	One-way trapdoor function/permutation	Factoring problem RSA-problem		Textbook RSA
Public-key encryption	Encrypt variable-length input $\operatorname{Enc}: \mathcal{PK}{\times}\mathcal{M} \to \mathcal{C}$	Decisional Diffie-Hellman (DDH) Factoring problem RSA-problem	IND-CPA IND-CCA	ElGamal Padded RSA
Digital signatures	Sign : $\mathcal{SK} \times \mathcal{M} \to \mathcal{S}$ Vrfy : $\mathcal{VK} \times \mathcal{M} \times \mathcal{S} \to \{1,0\}$	RSA-problem Discrete logarithm (DLOG)	UF-CMA	Hashed-RSA ECDSA Schnorr

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Assignment 1 (Deadline 10 March)

- Implement the ElGamal Enc algorithm in Sage
 - submit the code
 - Provide "known answer-test" (KAT) values (i.e., example of pk, sk, m and c)
- Implement the Textbook RSA signature in Sage
 - submit the code
 - And show the attack that if $\sigma_1=M_1^d$, $\sigma_2=M_2^d$, then σ_1 σ_2 is the Textbook RSA signature of M_1 M_2
 - Provide "known answer-test" (KAT) values (i.e., example of vk=(n, e), sk=d, m and σ)
- Write a report about the algorithms and implementation
- Assignment 1 will be available on the blackboard

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Network security

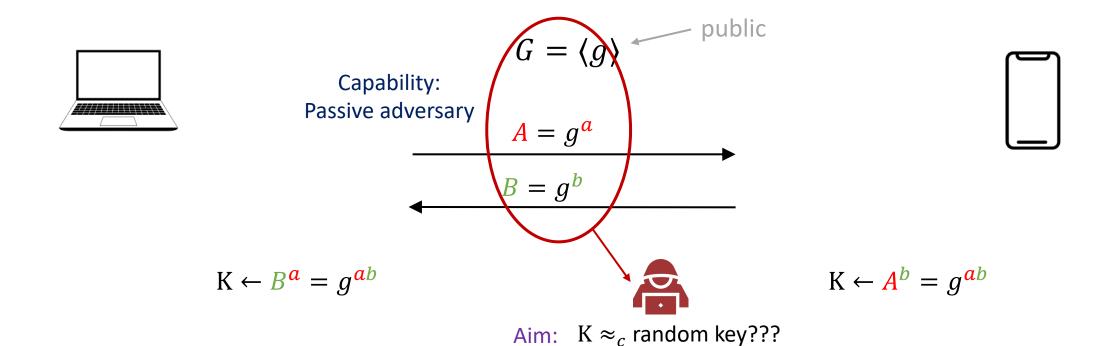
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authenticated key exchange

• public key infrastructure (PKI)

and certification authorities

Diffie-Hellman Key Exchange



Security (given G, g, A, B):

• Must be hard to distinguish $K \leftarrow g^{ab}$ from random key

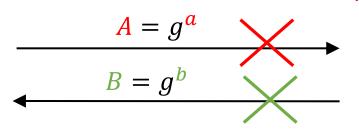
Diffie-Hellman Key Exchange



$$G = \langle a \rangle$$

Capability:

Active adversary?





 $K \leftarrow B^{\mathbf{a}} = g^{\mathbf{a}b}$



$$K \leftarrow A^b = g^{ab}$$

Aim: $K \approx_c$ random key???

Diffie-Hellman: man-in-the-middle attack









 $a \stackrel{\$}{\leftarrow} \{1, \dots, |G|\}$

$$B = g^b$$

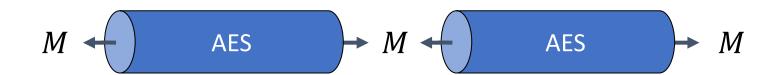
$$b \stackrel{\$}{\leftarrow} \{1, \dots, |G|\}$$

$$-\mathbf{K} \leftarrow \mathbf{A}^b = g^{ab}$$

$$K'' \leftarrow X^b = g^{xb}$$

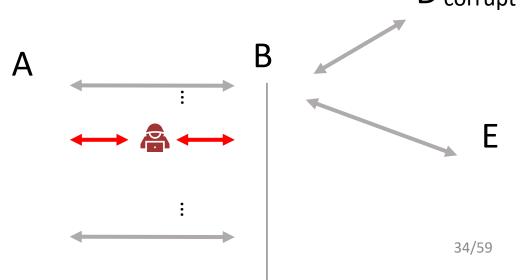
$$K' \leftarrow A^y = g^{ay}$$

$$\mathbf{K'} \leftarrow \mathbf{A}^{y} = g^{\mathbf{a}y} \qquad \qquad \mathbf{K''} \leftarrow B^{x} = g^{xb}$$



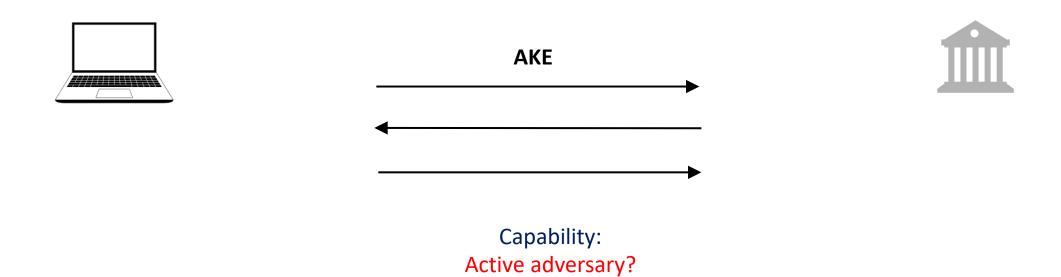
Active Adversary

- Adversary has complete control of the network:
 - Can modify, inject and delete packets
 - Like the man-in-the-middle attack
- Moreover, some internet users are honest and others are corrupted
 - Corrupt users are controlled by the adversary
 - Key exchange with corrupt users should not "affect" other sessions



Authenticated Key Exchange (AKE)

- key exchange secure against active adversaries
- AKE protocol should allow two users to establish a shared key, and ensure that they are talking with whom they plan to talk with

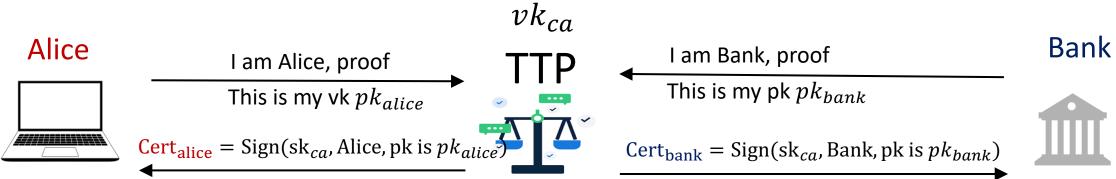


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Trusted Third Party

All AKE protocols require a TTP to certify user identities.

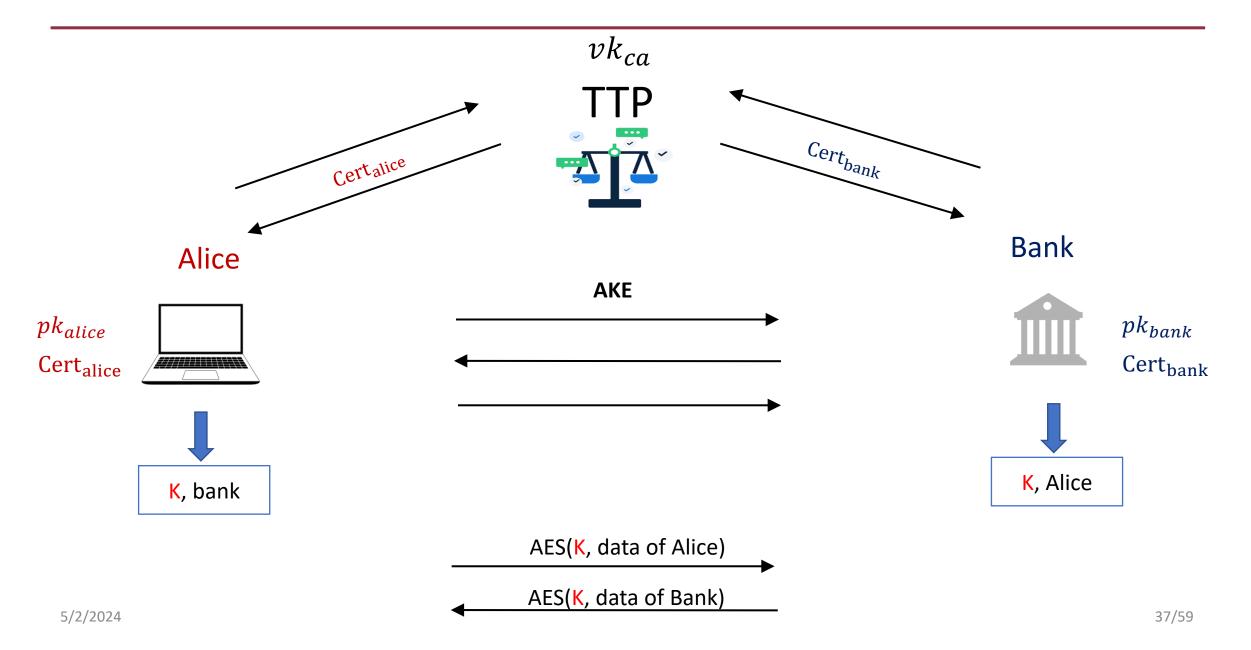
Registration process:





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AKE-syntax



Basic AKE security (very informal)

Suppose Alice successfully completes an AKE to obtain (K, Bank)

- If Bank is not corrupt then:
 - Authenticity for Alice: (similarly for Bank)
 - If Alice's key K is shared with anyone, it is only shared with Bank
 - **Secrecy** for Alice: (similarly for Bank)
 - To the adversary, Alice's key K is indistinguishable from random (aim)

Consistency: if Bank completes AKE then it obtains (K, Alice)

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Three levels (core) security of AKE:

• Static security: previous slide

• Forward secrecy: static security, and if the adversary learns sk_{bank} at time T then all sessions with Bank before T remain secure.

• Hardware Security Module (HSM): Forward secrecy, and if adversary queries an HSM holding sk_{bank} n times, then at most n sessions are compromised.

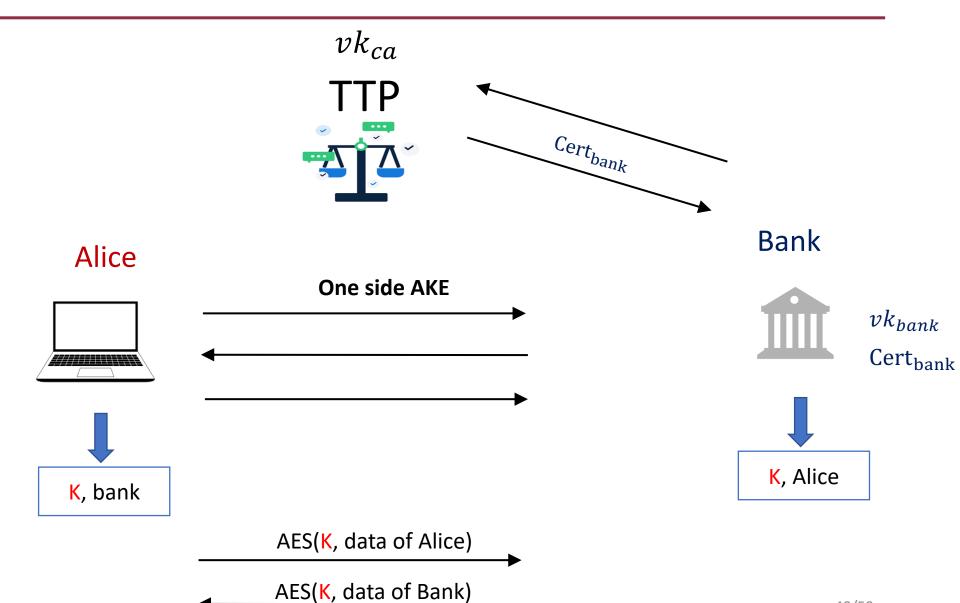
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One-sided AKE: syntax

only one side has a certificate

• three security levels.



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Protocol #1 Building blocks

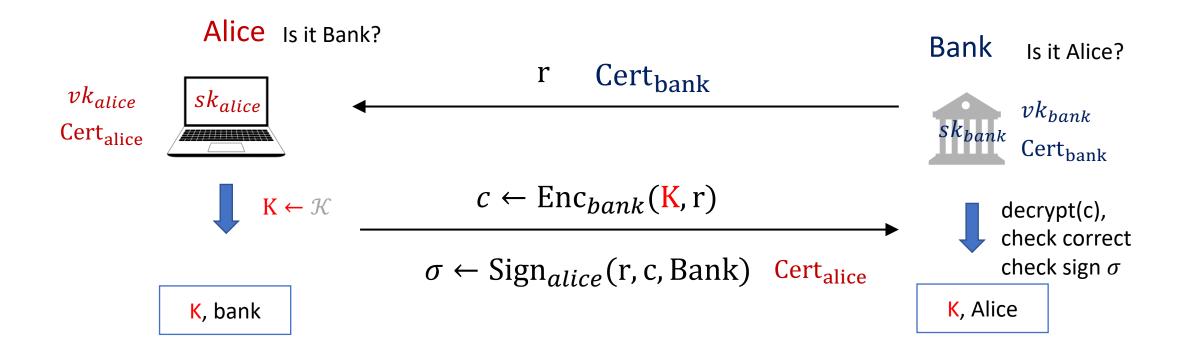
• Bank has $Cert_{bank}$ contains pk_{bank}

• ${
m Enc}_{bank}$: IND-CCA secure PKE using Bank's public key Bank keeps sk_{bank} as the secret encryption key

• $Sign_{alice}$ / $Sign_{bank}$: UF-CMA secure signature of Alice/Bank

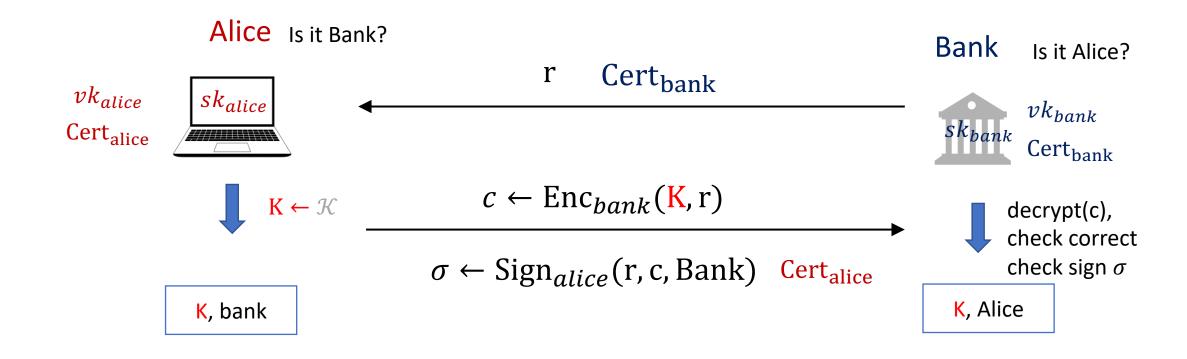
AES encryption scheme

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- Theorem: Protocol #1 is a statically secure AKE
- Informally: if Alice and Bank are not corrupt then we have
 (1) secrecy for Alice\Bank and (2) authenticity for Alice\Bank

Protocol #1 problem: no forward secrecy



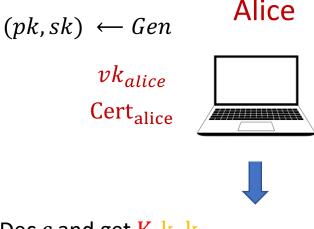
Suppose a year later adversary obtains sk_{bank} \Rightarrow can decrypt all recorded traffic Protocol #1 is used in TLS 1.2 not TLS 1.3

Protocol #2: HSM Security

Forward secrecy, and n queries to HSM should compromise at most n sessions

AKE4 of section 21.2 in A Graduate Course in Applied Cryptography

Protocol #2



Dec c and get K, k₁k₂

Dec c_1 with k_1 Check Sign

Delete sk

K, bank



$$c = Enc(pk, (K, k_1, k_2))$$

$$c_1 = AES(k_1; Cert_{bank}, Sign_{bank}(pk, c))$$

$$c_2 = AES(k_2; Cert_{alice}, Sign_{alice}(pk, c))$$

Bank Is it Alice?





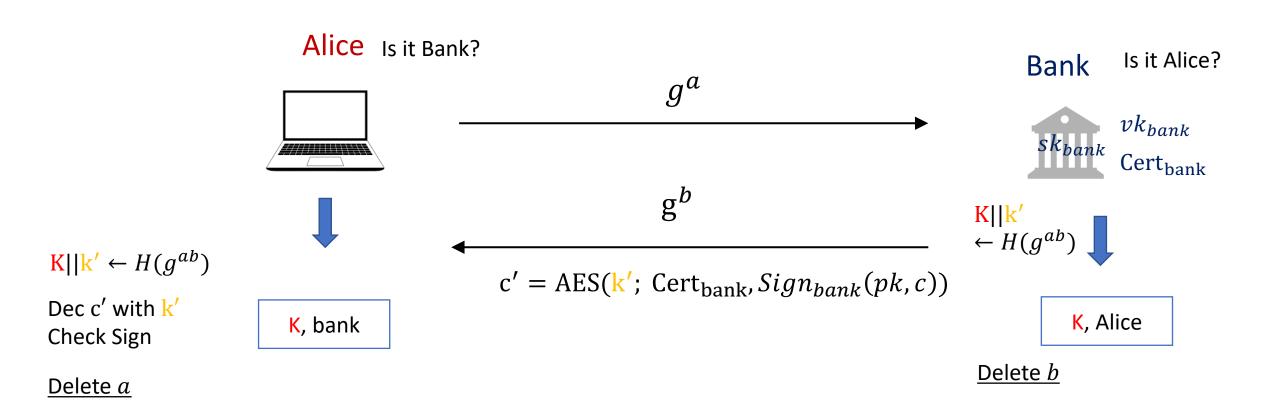
$$\begin{aligned} \mathbf{K} &\leftarrow \mathcal{K} \\ k_1 || k_2 &\leftarrow \mathcal{K} \end{aligned}$$

Dec c_2 with k_2 Check Sign

K, Alice

Main point: need to sign ephemeral pk from client ⇒ past access to HSM will not compromise current session

Protocol #4 one side-use Diffie-Hellman instead of PKE



[variant of TLS 1.3]

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A short summary

AKE requires TTP to certify user identities

Security: static security, Forward secrecy, HSM secrecy

• We can build secure AKE via PKE, signature, and/or, AES

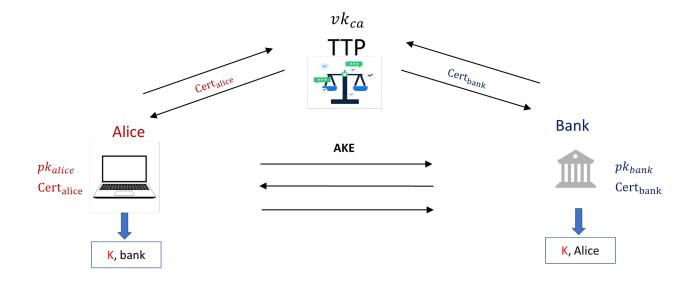
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Problem: public key infrastructure (PKI)

A single TTP

- Single point of failure
 - What if TTP is corrupted?

- How should we deploy the trust of certification?
 - How does Bank communicate with TTP to get Cert_bank?



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TTP: Certification Authorities

Digital Certification

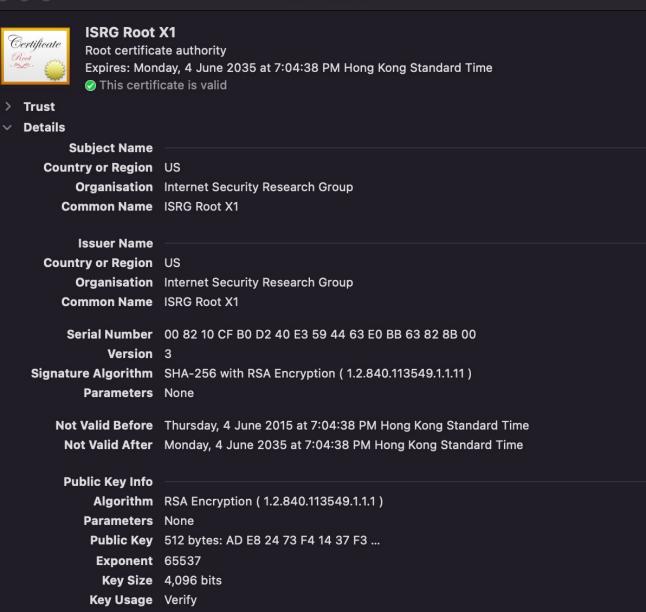


 $Cert_{bank} = Sign(sk_{ca}, Bank's public (sign) key is <math>vk_{bank}$; URL ishttps://www.hangseng.com/)

Any one with vk_{ca} can verify the $\operatorname{Cert}_{bank}$

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- Subject Name
 - Who's CA
- Issuer Name
 - Who gives this CA
 - Sign name
 - Valid
- PK information
 - pk
 - What is the pk is used
 - Key size



Signature 512 bytes: 55 1F 58 A9 BC B2 A8 50 ...

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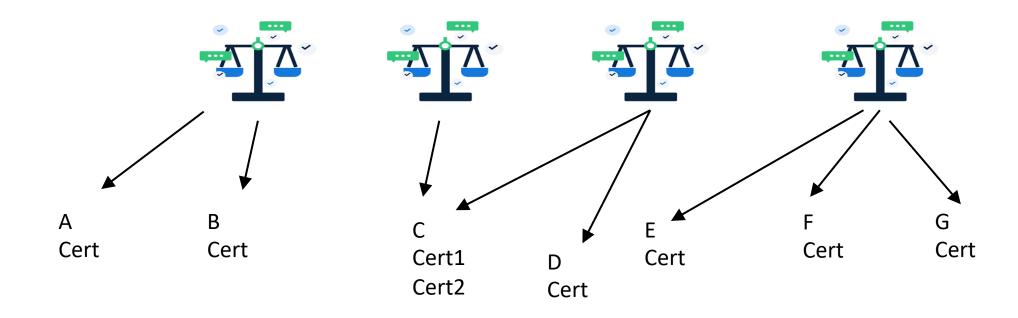
Certification Authorities(CA)



- How should I get the vk_{ca} of TTP?
- a root CA's public key is provided together with the browser/System

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Multiple CAs

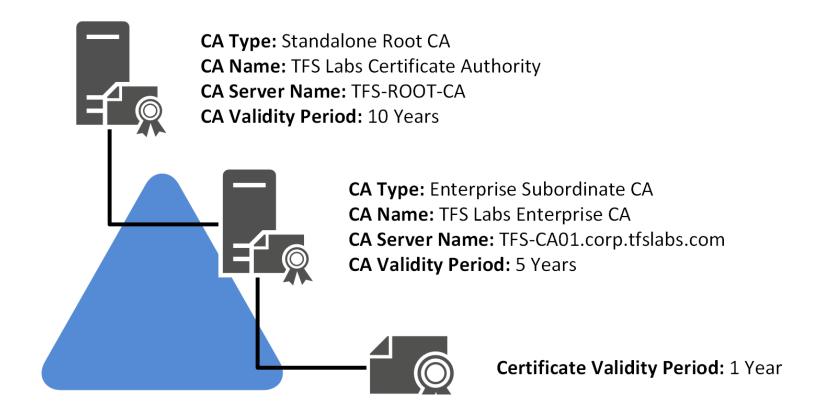


• Reduce the risk of single point of failure

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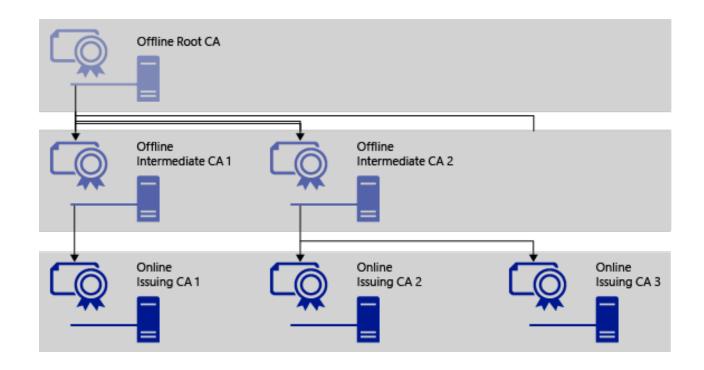
Authentication Chain

We could build the trust of certificate chains from a single Root CA



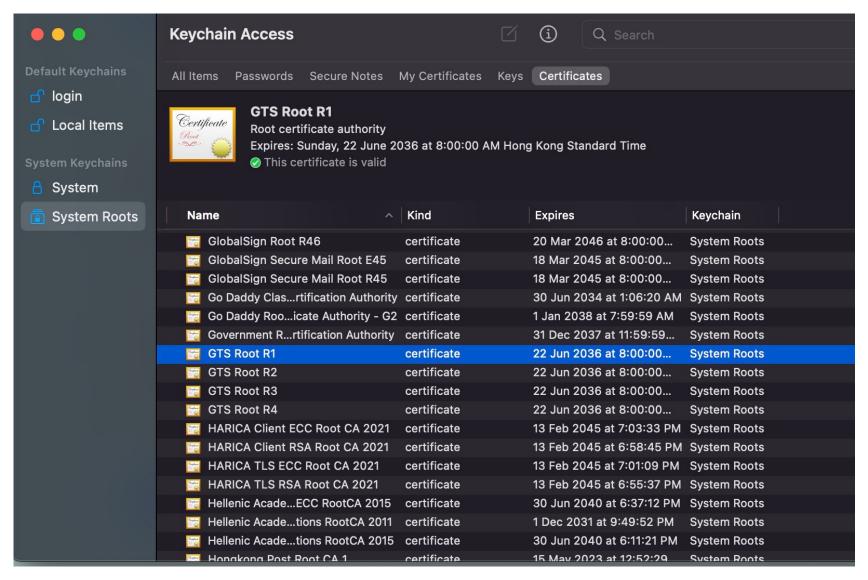
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Authentication Chain



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Root CA in Mac OS



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Root CA in Windows

Root CA in windows

 Select Run from the Start menu, and then enter certlm.msc. The Certificate Manager tool for the local device appears.

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Root CA in web browser

chrome://settings/security

Firefox

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Summary

- Recall RSA and Digital Signature
- Authenticated Key Exchange
- Public Key Infrastructure(PKI)
- and Certification Authorities
- For your lecture notes, please refer to
- [KL] Section 12.7
 Dan Boneh and Victor Shoup, <u>A Graduate Course in Applied Cryptography</u>, Section 22
 [Du] Section 24

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Thank you Happy Chinese New Year