Efficient MtA from Joye-Libert and Its Application to Threshold ECDSA

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PADO

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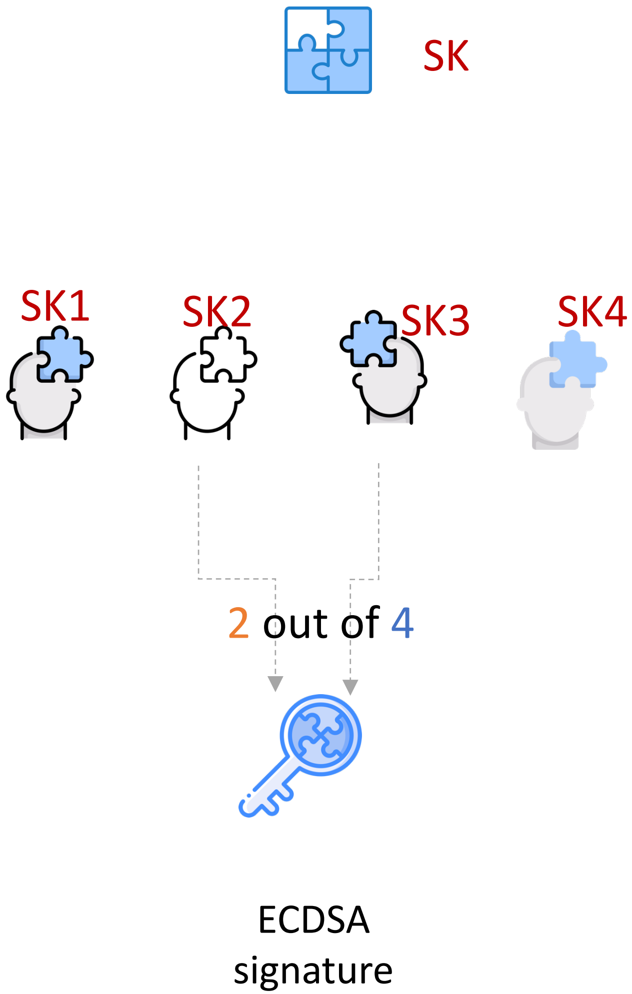
"Threshold digital signature"[DF89] aims to address this problem

[DF89] Yvo Desmedt and Yair Frankel. Threshold cryptosystems. In CRYPTO 1989

Threshold ECDSA signature

Goal: An ECDSA signature that looks like it was produced
by a single party, yet the key is stored in shared form

Naïve idea: divide key into shares



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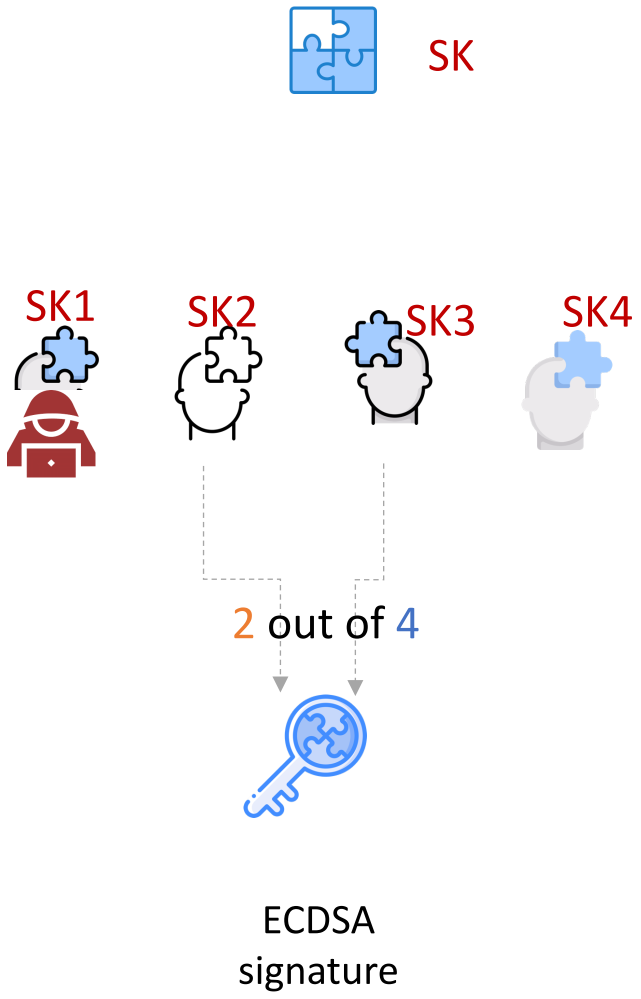
Even t-1 parties are compromised, Still Secure!



More about ECDSA

Input: secret key: $\mathrm{SK} = x$, message m

Output: Output (r, s) where

$$s = k^{-1}(H(m) + x \cdot r),$$

k is the randomness,
$H(m)$ is the hash of message m,
r can derived from $k \cdot P$

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- $H(m)$ and r can be public
- k and x should be kept secret

Challenge of Threshold ECDSA

Input: secret key: $\mathrm{SK} = x$, message m

Output: Output (r, s) where

$k_1, k_2, \cdots, k_t$ $\quad$ $x_1, x_2, \cdots, x_t$

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Challenge When Thresholding

How to securely compute (**non-linear**) k^{-1} and $k^{-1}x$
from shares of x and k

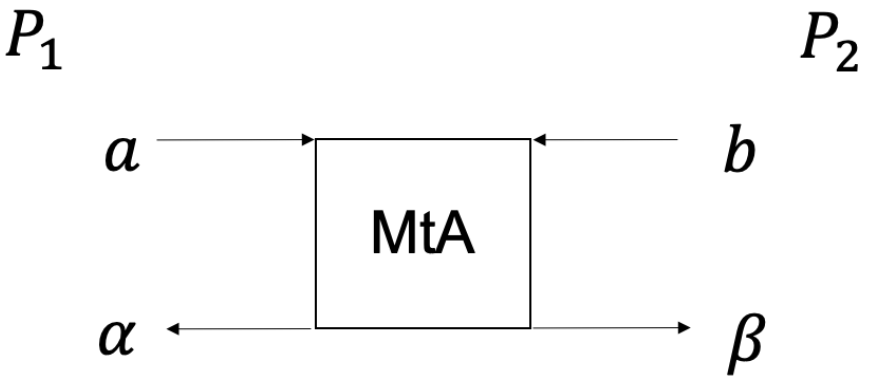
Even worse, some of the parties are controlled by the adversary

Main Building Block: MtA

Several works have been done to address the challenge

[GG18], [LN18], [DKLs18], [DKLs19], [CCL+19], [CCL+20], [CGG+21], [XAX+21], …

Multi-to-Add (MtA):



such that $\alpha + \beta = a \cdot b \bmod q$

State-of-the-art: MtA

MtA is costly

Tools ($\lambda = 128$)	Computation (ms)	Communication (KB)
Oblivious Transfer (OT) [DKLs18-19]	~10	90
Castagnos-Laguillaumie Enc [CCL+19,20]	~1600	~2
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Our goal is to find better candidates of MtA (than Paillier in both comp. and comm.)?

Our work: Better MtA from Joye-Libert

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Our work Joye-Libert	~200	4.1

When $\lambda = 192, 256$, the improvement over Paillier-base MtA achieves 48% (resp. 44%) in communication (resp. computation)

MtA from additive HE (semi-honest)

P_1 $\qquad\qquad\qquad\qquad$ P_2

a $\qquad\qquad\qquad\qquad$ b

$$\xleftarrow{\mathrm{C}_2 = \mathrm{Enc}\,(b)}$$

$$\alpha \quad \xrightarrow{\mathrm{C}_1 = \mathrm{Enc}\,(a \cdot b - \alpha)} \quad \beta = \mathrm{Dec}\,(C_1)\; mod\; q$$

Such that $\alpha + \beta = a \cdot b \bmod q$

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Alpha-rays attack [TS21] due to the mismatch of message space (Paillier ~3096 bits) and q (ECDSA~256 bits)

[TS21] Dmytro Tymokhanov and Omer Shlomovits. Alpha-rays: Key extraction attacks on threshold ecdsa implementations, eprint 2021.

MtA from Paillier

Schemes	over	(Message) Space
ECDSA	$mod\ q$ (~256 bits)	$mod\ q$ (~256 bits)
Paillier	$mod\ N^2$	$mod\ N$

Mismatch

To guarantee security (e.g. against Alpha-rays Attack [TS21])

- P_2 needs to prove: $R_2 = \{C_2; b \mid C_2 = Enc(b), b \in [0, q]\}$
- P_1 needs to prove: $R_1 = \{C_1; a, \alpha \mid C_1 = Enc(ab - \alpha), a, \alpha \in [0, q]\}$

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zero-knowledge range proof

Simple Observation:

There is waste message space in Paillier

Expensive zero-knowledge range proof is required

MtA from Joye-Libert Enc

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[BSJL17] Fabrice Benhamouda, Javier Herranz Sotoca, Marc Joye, and Benoit Libert. Efficient cryptosystems from $2k$-th power residue symbols. Journal of cryptology 2017

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The challenge is

no standard zero-knowledge (range) proof for Joye-Libert with large challenge space

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- Small challenge space, $e, e' \in \{0, 1\}$; inefficient
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- Standard soundness is based on the strong-RSA assumption

More Applications

- Three-party TLS handshake
  - for the Decentralized Oracle authenticating TLS data
- Naor-Yung CCA secure encryption
  - Two Joye-Libert Encs with zero-knowledge Range Proof
- Multiparty Computation (MPC)
  - SPDZ_2^k

Open Problems

- Efficient proof on the correctness of Joye-Libert modulus
  - $N = (2^k p' + 1)(2^k q' + 1)$
- More applications of our MtA
- Other candidate of MtA?

Summary

- Another choice of MtA from Joye-Libert
  - besides those based on Paillier, CL, and Oblivious Transfer
- Applications
  - Threshold ECDSA, Naor-Yung CCA, Three-party TLS, etc.
- Zero-knowledge range proof for Joye-Libert cryptosystem

Eprint: 2023/1312

Thanks

Q & A